MODELING OF RADIATION-CONVECTION HEATING OF COHESIVE-SOIL GRANULES IN LOW-TEMPERATURE PLASMA FLOWS

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One way to maintain the rate of construction of automobile roads in regions which do not have rock-materials deposits is to use thermally stabilized soils in layers of the ground bed and road toppings [1]. Under the conditions of linear construction of automobile roads in regions where transportation routes are not adequately developed, the production bases are characteristically spread out and far away. In such a situation the technology associated with stabilizing pregranulated and dried local cohesive soils in reactors equipped with electric-arc plasmatrons [2] is economically justified.

In order to work out the technological aspects of the production of artificial material in eletroplasma setups it is important to determine the optimal sizes of the soil particles fed into the reactor as well as the temperature that would ensure transformation of the composition and properties of the raw materials during the short periods of thermostabilization processing. Calculations of convective heat transfer in cohesive-soil particles that take into account the radial temperature distribution in a sphere were performed by the numerical method described in [3]. Analysis of the computational results enabled determining the optimal conditions for the technology of thermostabilization of soils and working out the design concept of an electroplasma setup for processing many tons of coarsely dispersed raw material. However, high-temperature processes for treating granules in gas flows generated by electric-arc plasmatrons are possible under the combined action of radiation and convection heat transfer. For this reason, in the present paper we attempt to take into account radiation–convection heat transfer during heating of a spherical soil particle in a low-temperature plasma flow.

In this case the boundary-value problem describing heat transfer between the high-temperature gas flow and a spherical particle has the following form in the dimensionless representation:

$$\frac{1}{\eta^2}\frac{\partial}{\partial\eta}\left(\eta^2\lambda(\Theta)\frac{\partial\Theta}{\partial\eta}\right) = R^2\rho c\frac{\partial\Theta}{\partial t}, \quad 0<\eta<1, \quad t>0; \tag{1}$$

$$\Theta(\eta, 0) = \Theta_0(\eta), \quad \lambda\frac{\partial\Theta}{\partial\eta} = 0, \quad \eta = 0; \tag{2}$$

$$\lambda\frac{\partial\Theta}{\partial\eta} + R\alpha(\Theta - \Theta_f) + R\sigma T_*^3(\Theta^4 - \Theta_f^4) = 0, \quad \eta = 1. \tag{3}$$

Here $\eta = r/R$; $\Theta = T/T_*$; $\Theta_0 = T_0/T_*$; $\Theta_f = T_f/T_*$; T and T_0 are the instantaneous and initial particle temperatures; T_f is the temperature of the gas medium; and r and R are the radii of the particle.

We introduce the thermal conductivity function $u(\Theta) = \int_\omega^\Theta \lambda(z)dz$ and subtract u(Θ) from both sides of Eq. (4). Then the differential operator on the left-hand side of Eq. (1) becomes linear and the problem (1)-(3) assumes the form

$$\frac{1}{\eta^2}\frac{\partial}{\partial\eta}\left(\eta^2\frac{\partial u}{\partial\eta}\right) - u = \frac{R^2\rho c}{\lambda(\Theta)}\frac{\partial u}{\partial t} - u, \quad 0<\eta<1, \quad t>0; \tag{4}$$

$$u(\eta, 0) = \int_\omega^{\Theta_0}\lambda(z)\,dz, \quad \frac{\partial u}{\partial\eta} = 0, \quad \eta = 0, \quad \frac{\partial u}{\partial\eta} = q(\Theta), \quad \eta = 1$$
$$(q(\Theta) = -R\alpha(\Theta - \Theta_f) - R\varepsilon\sigma T_*^3(\Theta^4 - \Theta_f^4)). \tag{5}$$

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In order to construct the Green's function $G(\eta, z)$ the boundary-value problem (4) and (5) can be written in the conjugate form. The homogeneous boundary-value problem from which the Green's function is determined will assume the form

$$\frac{\partial^2 G}{\partial \eta^2} - \frac{\partial}{\partial \eta}\left(\frac{2}{\eta} G\right) - G = \delta(\eta, z); \tag{6}$$

$$\left(\frac{\partial G}{\partial \eta} - \frac{2}{\eta} G\right)_{\eta=0} = 0, \quad \left(\frac{\partial G}{\partial \eta} - \frac{2}{\eta} G\right)_{\eta=1} = 0. \tag{7}$$

With the help of the fundamental solutions $y_1 = \eta \sinh \eta$ and $y_2 = \eta e^{\eta}$ of Eq. (6), each of which satisfies the corresponding boundary condition (7), we can construct the following formal expression for the Green's function [4]:

$$G(\eta, z) = \begin{cases} a(\eta)\, y_1(z), & z \leqslant \eta, \\ b(\eta)\, y_2(z), & z \geqslant \eta. \end{cases}$$

Here $a(\eta) = e^{\eta}/\eta$, $b(\eta) = \sinh \eta/\eta$ are solutions of the system of equations

$$a(\eta)\, y_1(\eta) - b(\eta)\, y_2(\eta) = 0, \quad a(\eta)\, y_1'(\eta) - b(\eta)\, y_2'(\eta) = -1,$$

characterizing the continuity and jump in the derivative at $z = \eta$ for the Green's function, which we express explicitly as

$$G(\eta, z) = \begin{cases} -z \operatorname{sh} z e^{\eta}/\eta, & 0 < z \leqslant \eta, \\ -z e^{z} \operatorname{sh} \eta/\eta, & \eta \leqslant z \leqslant 1. \end{cases} \tag{8}$$

With the help of the Green's function (8) we reduce the boundary-value problem (4) and (5) to an integral equation for the function $u(\Theta)$, multiplying Eq. (4) by $G(\eta, z)$ and integrating by parts in the interval (0, 1):

$$\int_0^1 \left\{\frac{\partial^2 G}{\partial z^2} - \frac{\partial}{\partial z}\left(\frac{2}{z} G\right) - G\right\} u(z)\, dz + G \frac{\partial u}{\partial z}\bigg|_0^1 - u(z) \left(\frac{\partial G}{\partial z} - \frac{2}{z} G\right)\bigg|_0^1 = \int_0^1 G(\eta, z)\, F(\Theta)\, dz. \tag{9}$$

The expression in braces in Eq. (9) is a delta function $\delta(\eta, z)$, so that

$$\int_0^1 \delta(\eta, z)\, u(z)\, dz = u(\eta).$$

With the boundary conditions (7) the expression in parentheses vanishes, which gives

$$u(\eta, t) = \int_0^1 G(\eta, z)\, F(\Theta)\, dz - G(\eta, 1) \left(\frac{\partial u}{\partial z}\right)_{z=1},$$

Substituting into this equation the values of $G(\eta, 1)$ from Eq. (8) and $\left(\frac{\partial u}{\partial z}\right)_{z=1}$ from Eq. (5) we obtain a nonlinear integral equation for the desired temperature $\Theta(\eta, t)$:

$$u(\Theta) = -G(\eta, 1)\, q(\Theta) + \int_0^1 G(\eta, z)\, F(\Theta)\, dz$$
$$\left(F(\Theta) = \frac{R^2 \rho c}{\lambda(\Theta)} \frac{\partial u}{\partial t} - u(\Theta)\right). \tag{10}$$

With the help of the expression for $u(\Theta)$ and $q(\Theta)$ we can put the Eq. (10) into the form

$$\int_\omega^\Theta \lambda(z)\, dz - G(\eta, 1)\, \{R\alpha(\Theta - \Theta_f) + R\varepsilon\sigma T_*^3 (\Theta^4 - \Theta_f^4)\} = \int_0^1 G(\eta, z) \left\{R^2 \rho c \frac{\partial \Theta}{\partial t} - \int_\omega^\Theta \lambda(y)\, dy\right\} dz. \tag{11}$$

The solution of the nonlinear integral Eq. (11) makes it possible to obtain the temperature distribution $\Theta(\eta, t)$ as a function of the radius in a spherical particles at any time t from the start of the heat treatment.

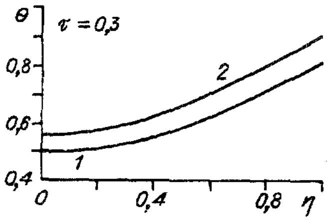

Fig. 1

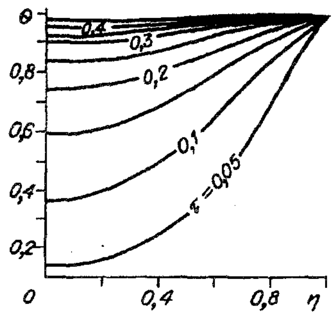

Fig. 2

The temperature dependence of the thermal conductivity of soil in the range 20-1200°C was determined experimentally and approximated by the least-squares method with a quadratic polynomial with respect to the dimensionless temperature $\Theta(\eta, t)$:

$$\lambda(\Theta) = a_0(\Theta - \omega)^2 - b_0(\Theta - \omega) + c_0$$
$$(a_0 = 0{,}985 \cdot 10^{-3},\ \ b_0 = 1{,}178,\ \ c_0 = 0{,}692,\ \ \omega = 0{,}219). \tag{12}$$

Substituting the expression for $\lambda(\Theta)$ from Eq. (12) into Eq. (11) we obtain

$$\Lambda(\Theta) + q(\Theta)\,G(\eta, 1) = \int_0^1 G(\eta, z)\left\{R^2\rho c\,\frac{\partial\Theta}{\partial t} - \Lambda(\Theta)\right\} dz$$
$$\left(\Lambda(\Theta) = \frac{a_0}{3}(\Theta - \omega)^3 - \frac{b_0}{2}(\Theta - \omega)^2 + c_0(\Theta - \omega)\right). \tag{13}$$

The iteration method described in [5] was used to solve numerically the nonlinear integral Eq. (13). The time derivative $\partial\Theta/\partial t$ was approximated by a finite-difference ratio, and the integral was calculated by Gaussian quadrature with 12 nodes. The radial temperature profile of the spherical soil particle was calculated after multiplying the left- and right-hand sides of Eqs. (13) by $1/\lambda_*$:

$$\Lambda(\Theta) - w(\Theta)\,G(\eta, 1) = \int_0^1 G(\eta, z)\left\{\frac{\partial\Theta}{\partial\tau} - \Lambda(\Theta)\right\} dz. \tag{14}$$

Here $w(\Theta) = \mathrm{Bi}(\Theta(1, \tau) - \Theta_f) + \varepsilon\mathrm{Sk}\,(\Theta^4(1, \tau) - \Theta_f^4)$; $\Lambda = \Lambda/\lambda_*$; $\mathrm{Bi} = \alpha R/\lambda_*$; $\tau = t/(R^2\rho c)$; $\mathrm{Sk} = \sigma R T_*^3/\lambda_*$; and λ_* is the characteristic thermal conductivity.

The numerical results of solving Eq. (14), which are presented in Figs. 1 and 2, were obtained with the following initial thermal-technical and optical characteristics: the thermal conductivity of the process soil $\lambda_* = 0.69$ W/mK, the temperature of the gas medium $T_* = 3000$ K, the soil emissivity $\varepsilon = 0.95$, the Biot number $\mathrm{Bi} = 0.94$, and the emitter temperature $\Theta_f = 0.9$. The calculations reflect the radial distribution of the dimensionless temperature Θ for the dimensionless coordinates and time τ. As follows from Fig. 1, in spite of the significant difference in the values of the radii (curves 1 and 2 correspond to $R = 20 \cdot 10^{-3}$ and $5 \cdot 10^{-3}$ m), the heat pulse moves from the surface to the center of a spherical soil particle with approximately the same speed R/τ. This is consistent with the theory of heat conduction [6], according to which for a spherical particle with minimum surface area per unit volume, the intensity of heating or cooling increases with the radius of the particle. We note that the computational results indicate that radiant heat transfer plays a significant role in the combined radiation–convection heating of soil particles.

Figure 2 displays the computed plot of the nonsteady temperature for a soil particle with a diameter of $15 \cdot 10^{-3}$ m. Such plots, constructed taking into account the specific size of the soil particles being stabilized, make it possible to trace the dynamics of the propagation of a heat pulse from the periphery to the center of the sphere ($r = 0$) for certain values of their initial temperature and the temperature of the gas medium as well as to analyze other aspects of the thermal process.

In conclusion we note that the intensity of heating of soil particles depends significantly on their initial temperature and the gas temperature in the working zone of the reactor. At the same time, under conditions of radiation–convection heat transfer, the size of the particles being stabilized plays a quite small role in the process of heating of the particles in low-temperature plasma flows. Further analysis of the results of modeling, taking into account the experimental data, will make it possible to refine the optimal conditions for the technology of thermostabilization of granules of cohesive soil.

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